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Two-dimensional natural convection and conduction in a packed bed containing a hot spot and its relevance to the transport of air in a coal dump

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Abstract—It is shown that experimentally-measured temperature profiles in a porous medium with heat generation along a line source can be modelled in terms of conduction and convection. The model predicts that convection always occurs for non-zero heat generation and that the concept of a critical Rayleigh number is not appropriate in this case. Using the model it is possible to calculate the volume of air flowing into the bed. It is concluded that natural convection is a feasible mechanism for oxygen transport into a coal dump where sufficient reaction may occur to sustain a hot spot.

INTRODUCTION

SPONTANEOUS heating of mined coal and the formation of so-called 'hot spots' within packed beds of stored coal particles has often been observed [1–4]. The intrinsic reactivity of a particular coal is an important factor, but irrespective of this reactivity, the hot spot will not be sustained if insufficient oxygen reaches it or if the heat generated by the reaction can be effectively transferred out of the bed. Similar situations exist in porous media with internal heat generation such as the underground disposal of nuclear waste [5] and the storage of agricultural products in closed containers [6].

In assessing factors concerning spontaneous combustion of coal some workers [7, 8] have considered conduction as the only heat transfer mechanism. This is based on the assumption that the onset of natural convection will only occur above a certain critical Rayleigh number, but this is only true for a limited class of problems [9, 10].

Beukema *et al.* [6] developed a model of three-dimensional natural convection in a confined porous medium with uniform internal heat generation. Experiments were performed on cooling the bed and they found that, compared with conduction only, natural convection caused enhanced cooling. Brooks [11] developed a one-dimensional model of a coal bed and concluded that natural convection is an important transport mechanism. It is the intention of this work to extend Brook's model to the two-dimensional case of mass and energy transport in the vicinity of a hot spot. The model permits calculation of the gas flowrate entering the bed which is important for calculating the amounts of reactants arriving at a hot spot and an assessment may be made as to whether this is sufficient to sustain its temperature.

MODEL EQUATIONS

The system to be modelled is shown diagrammatically in Fig. 1. This depicts a rectangular cross-section through the packed bed where the energy source is a small rod-like electrical element which passes through the bed at right-angles to the cross-section. In the derivation of the model, the two-dimensional equations for continuity, momentum and energy are simplified by neglecting inertia forces in the momentum equation and viscous heating by fluid flow in the energy equation. The Boussinesq approximation [12] is made and Darcy's model [13] is used to replace the shear stress tensor in the equation of motion. It is also assumed that the porous medium may be treated as homogeneous so that only one set of conservation equations need be written with conduction being modelled by a single effective thermal conductivity. These assumptions have been used by other workers in the field [9, 14] and have been experimentally justified [6, 10].

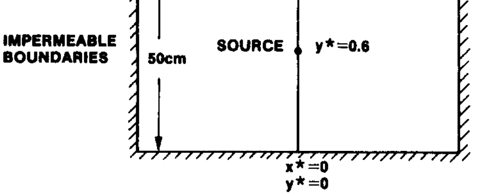

FIG. 1. Geometry of the bed showing position of heat source and boundaries.

NOMENCLATURE

C_p	heat capacity [J kg^{-1} K^{-1}]
D	characteristic dimension [m]
g	acceleration of gravity [m s^{-2}]
G	energy generation rate [W m^{-3}]
k_{eq}	effective thermal conductivity [W m^{-1} K^{-1}]
K	permeability [m s^{-1}]
L	distance coordinate [m]
n	molar flowrate of oxygen [mol s^{-1}]
Pe	Péclet number
q	source power [W m^{-1}]
Q	air flowrate [m^3 s^{-1}]
T	temperature [K]
v	velocity [m s^{-1}]
x, y	distance coordinates [m].

Greek symbols

δ	distance coordinate [m]
ΔH_r	enthalpy of reaction of oxygen with coal [kJ mol^{-1}]
ρ	density [kg m^{-3}]
ψ	streamfunction.

Subscripts

0	ambient conditions
g	gas phase
x	x-component
y	y-component.

Superscripts

1	deviation from ambient conditions
*	dimensionless variable.

With these assumptions the governing equations under steady-state conditions are [15]:

equation of continuity

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

equation of motion

$$-\nabla P - \frac{\rho_0 g_0}{K}\mathbf{v} + \rho_g \cdot \mathbf{g} = 0 \tag{2}$$

equation of energy

$$\rho_g C_{pg} \mathbf{v} \cdot \nabla T = k_{eq} \nabla^2 T + G. \tag{3}$$

The equations are made dimensionless by the transformations

$$x^* = x/D, \quad y^* = y/D$$

$$v_x^* = v_x/K, \quad v_y^* = v_y/K$$

$$P^* = \frac{P^1}{\rho_0 K^2}, \quad T^* = \frac{T - T_0}{T_0}$$

where

$$P^1 = P - P_0 - \rho_0 g(H - y) \tag{4}$$

defined in this way so that the dimensionless pressure is zero in a stagnant bed.

The gas density is written as:

$$\rho_g = \rho_0 + \rho^1. \tag{5}$$

The density is assumed to be a function of temperature only since pressure variations will be small. The gas is assumed ideal and thus to a first-order approximation

$$\rho^1 \simeq -\rho_0 T^*. \tag{6}$$

The experimental apparatus used corresponds to the two-dimensional system shown in Fig. 1 and the problem can thus be simplified by introducing the streamfunction ψ which is defined by:

$$\begin{aligned} v_x^* &= -\frac{\partial \psi}{\partial y^*} \\ v_y^* &= \frac{\partial \psi}{\partial x^*}. \end{aligned} \tag{7}$$

This definition of the streamfunction automatically satisfies the continuity equation. Substitution of (6) and (7) into the equation of motion, and the elimination of P yields a Poisson equation in the streamfunction

$$\frac{\partial^2 \psi}{\partial x^{*2}} + \frac{\partial^2 \psi}{\partial y^{*2}} = \frac{\partial T^*}{\partial x^*}. \tag{8}$$

In previous work [9, 10] the RHS of (8) was zero and ψ zero everywhere was a solution. However, when there are large temperature gradients in all directions, as is the case with a hot spot, convection will always occur.

The equation in the streamfunction (8) must be solved simultaneously with the equation of energy, which in dimensionless form is

$$\frac{1}{Pe}\left[v_x^* \frac{\partial T^*}{\partial x^*} + v_y^* \frac{\partial T^*}{\partial y^*}\right] = \frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{GD^2}{k_{eq} T_0} \tag{9}$$

where Pe is the Péclet number, defined as

$$Pe = \frac{\alpha}{DK}, \quad \text{where} \quad \alpha = \frac{k_{eq}}{\rho_g C_{pg}}.$$

In the limit as the Péclet number becomes large, conduction dominates and the equations are decoupled. At low Péclet numbers, however, convection becomes important and equations (8) and (9) are strongly linked.

The experimental bed as shown in Fig. 1 is symmetrical and therefore the model equations need only be solved in the half of the rectangle bounded by $0 \leqslant x^* \leqslant 1$ and $0 \leqslant y^* \leqslant 1$. The boundary conditions on the streamfunction (8) for the situation as shown in Fig. 1 are:

$$\psi(0, y^*) = 0 \qquad (10)$$

$$\psi(1, y^*) = 0 \qquad (11)$$

$$\psi(x^*, 0) = 0 \qquad (12)$$

$$\frac{\partial \psi}{\partial y^*}(x^*, 1) = 0. \qquad (13)$$

The boundary conditions for the equation of energy (9) are based on the experimental observation that to a fair approximation the boundaries are at ambient temperature. This is found to be the case even directly above the source. Thus the following conditions are used:

$$\frac{\partial T^*}{\partial y^*}(0, y^*) = 0 \qquad (14)$$

$$T^*(1, y^*) = 0 \qquad (15)$$

$$T^*(x^*, 0) = 0 \qquad (16)$$

$$T^*(x^*, 1) = 0. \qquad (17)$$

Equations (8) and (9) are solved using an alternating direction implicit procedure [16].

EXPERIMENTAL

A steel container was constructed as shown in Fig. 2. The lower section of the container was filled with coal of roughly 1-cm particle diameter. The experiments were run in a nitrogen atmosphere to minimize oxidation of the coal. The hot spot was simulated by passing known amounts of power through the rod element in the bed. The temperature at different locations in the bed was measured with thermistors using an automated system. With this system 45 different temperatures could be measured at preselected points in the bed as shown in Fig. 3.

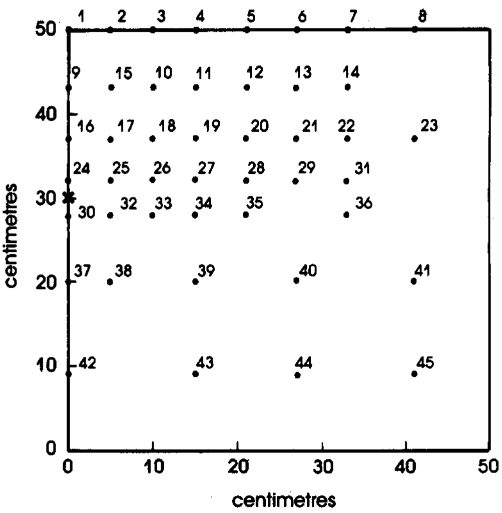

FIG. 3. Position of thermistors in the experimental apparatus.

Before each experiment, nitrogen was blown into the experimental apparatus to cool the bed to ambient temperature and replenish any nitrogen that may have been lost. The element was then turned on, and the power output of the element controlled. The experiments were stopped after about a week when steady state had been attained. These experiments were run at a number of different source powers.

Application of the model to the experiments

In the model there are essentially two parameters: the effective thermal conductivity, which has been measured independently [11]; and the permeability, which can be roughly estimated using Ergun's equation [17]. The model is fairly sensitive to these parameters and it was decided to fit the parameters to the

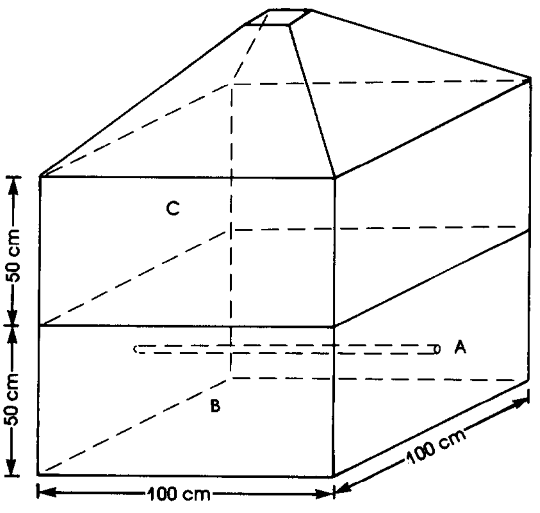

FIG. 2. Diagram of experimental apparatus: A, heating element; B, section packed with coal; C, open section filled with nitrogen.

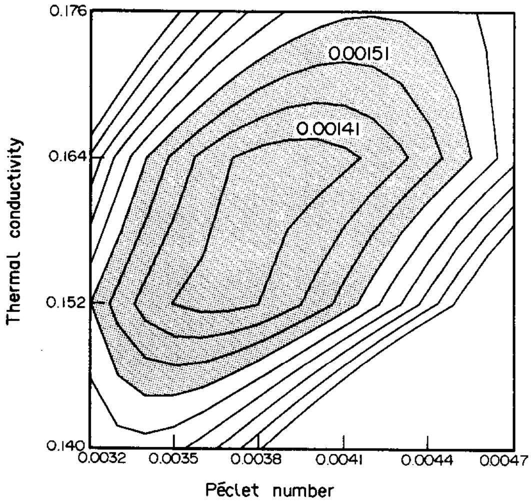

FIG. 4. Typical set of least-squares contours with the 90% confidence region shaded.

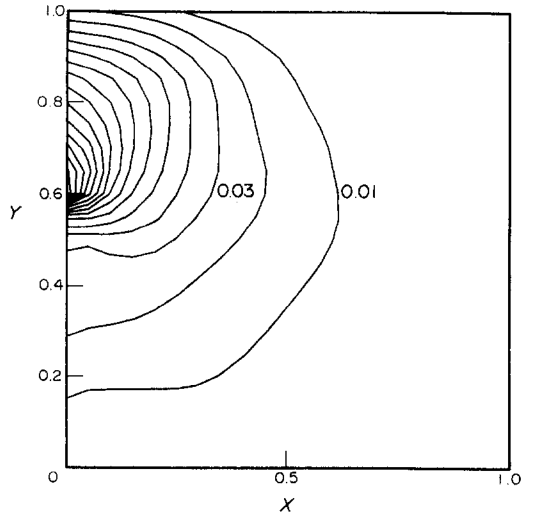

FIG. 5. Measured temperature profile, lines drawn for intervals of 0.01 in T^*.

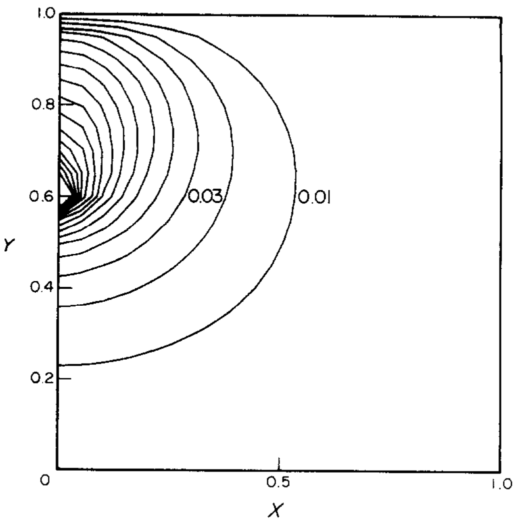

FIG. 6. Model-predicted temperature profile.

experimental data, and then check that the fitted values are reasonable.

The parameters were fitted by minimizing the sum of squared errors between the measured temperatures and those predicted by the model, by choice of the Péclet number and the thermal conductivity. Confidence intervals were obtained for these parameters, the significance of which for non-linear models is discussed in some detail by Box *et al.* [18].

Contours of the least-squares surface for one of the experiments are presented in Fig. 4. The 90% confidence region for the parameters is shaded. These parameters values are not significantly different from independent determination of the parameters.

Contours of an experimentally-determined temperature profile are presented in Fig. 5 and a model temperature profile is shown in Fig. 6 for comparison. In order to get a better idea of the agreement, results are plotted along vertical slices in the bed and these results are presented in Fig. 7. Distortion of the temperature profile from that which would have been expected had there been conduction only, is clearly visible.

The type of flow predicted by the model is indicated by the streamlines in Fig. 8. Typically, gas flows in at the top of the container and flows out above the source, with some recirculation of gas. As the source power is increased so the effect of convection becomes more marked, whereas at low power conduction dominates

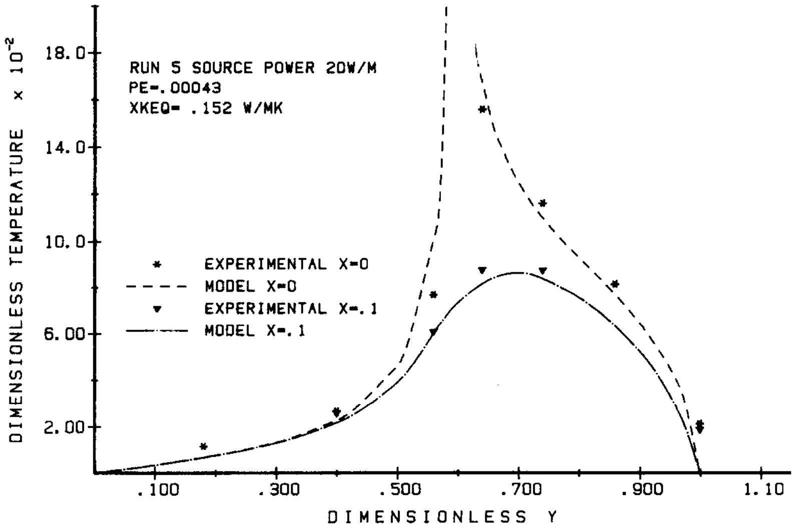

FIG. 7. Comparison of temperature profile for model and experimental results along a vertical slice through the bed.

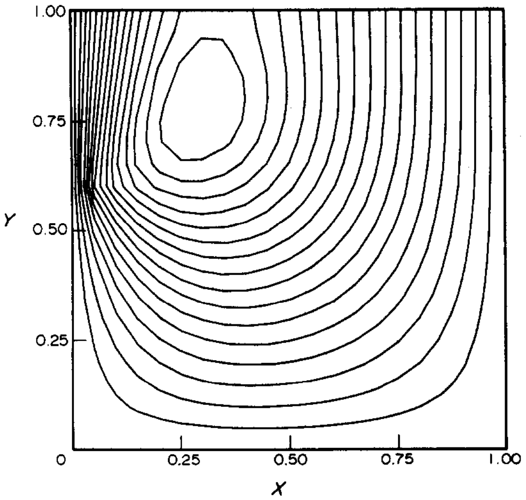

FIG. 8. Streamlines predicted by the model.

and consequently poor estimates of permeability will be obtained in the latter case.

CONCLUSIONS

Since the measured and model temperature profiles agree very well, it can be concluded that the model gives an adequate prediction of the velocity field. From this it is clear that convection always occurs for non-zero heat generation. The concept of a critical Rayleigh number is therefore not appropriate in this case.

An objective of this work is to determine whether natural convection plays a significant role as a mechanism by which oxygen can reach and react at a hot spot in a coal bed. A rough calculation shows that an air flowrate per unit length of rod of 1×10^{-6} m^2 s^{-1} will sustain a hot spot generating 5 W m^{-1} with a ΔH_r of approx. 400 kJ mol^{-1} [19]. The mathematical model with thermal conductivity of 0.16 W m^{-1} K^{-1} and Péclet number of 4×10^{-3} predicts that, for a source power of 5 W m^{-1}, 2×10^{-4} m^2 s^{-1} of air will flow into the bed. Natural convection is thus likely to be an important phenomenon in the study of coal oxidation in dumps.

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CONVECTION NATURELLE BIDIMENSIONNELLE ET CONDUCTION DANS UN LIT FIXE CONTENANT UN POINT CHAUD ET APPLICATION AU TRANSFERT D'AIR DANS UN LIT DE CHARBON

Résumé—On montre que les profils de température expérimentaux dans un milieu poreux avec une génération de chaleur le long d'une ligne peuvent être modélisés par la conduction et la convection. Le modèle prédit que la convection existe toujours pour une génération de chaleur non nulle et que le concept d'un nombre de Rayleigh critique n'est pas approprié dans ce cas. En utilisant le modèle il est possible de calculer le volume d'air traversant le lit. On conclut que la convection naturelle est un mécanisme possible pour le transfert d'oxygène dans une charge de charbon où une réaction est suffisante pour maintenir un point chaud.

ZWEIDIMENSIONALE NATÜRLICHE KONVEKTION UND WÄRMELEITUNG IN EINEM FESTBETT MIT HEISSEN STELLEN—BEDEUTUNG FÜR DEN LUFTTRANSPORT IN KOHLE

Zusammenfassung—Es wird gezeigt, daß experimentell gemessene Temperaturprofile in einem porösen Medium mit linienförmiger Wärmequelle mit Hilfe von Leitungs- und Konvektionsgleichungen beschrieben werden können. Das Modell sagt voraus, daß bei beliebig kleiner Wärmezufuhr Konvektion auftritt. In diesem Fall ist das Konzept der kritischen Rayleigh-Zahl nicht anwendbar. Mit dem Modell kann der Luftvolumenstrom berechnet werden, der in das Bett strömt. Daraus kann man schließen, daß die natürliche Konvektion der grundsätzliche Mechanismus für den Sauerstofftransport in einem Kohleklumpen ist, in welchem ausreichende Reaktionen stattfinden, um eine heiß Zone aufrechtzuerhalten.

ДВУМЕРНАЯ СВОБОДНАЯ КОНВЕКЦИЯ И ТЕПЛОПРОВОДНОСТЬ В ПЛОТНОМ СЛОЕ, СОДЕРЖАЩЕМ ГОРЯЧУЮ ОБЛАСТЬ, ПРИМЕНИТЕЛЬНО К ПЕРЕНОСУ ВОЗДУХА В УГОЛЬНЫХ ОТВАЛАХ

Аннотация—Показано, что экспериментально измеренные профили температуры в пористой среде с линейным источником тепла могут быть смоделированы механизмами конвекции и теплопроводности. Эта модель предсказывает, что конвекция всегда возникает при отличном от нуля тепловыделении, и что понятие критического числа Рэлея не применимо в данном случае. Используя предложенную модель, можно рассчитать объем воздуха, проникающего в слой. Сделан вывод о том, что естественная конвекция является возможным механизмом процесса, переносящим кислород в угольном отвале, в котором может начаться реакция с выделением тепла.